Unit-I: Sets
Short Answer Questions:
Q. Define Power Set with an illustration. (Nov 22),(Nov 23)
Ans. A **power set** is the set of all subsets of a given set.
Example: If A = {1, 2}, then P(A) = {∅, {1}, {2}, {1, 2}}.

Q. If A, B are any sets, then A ∩ B = B if B ⊂ A. (Nov 22)
Ans. If set B is a subset of set A, their intersection A ∩ B equals B.
This is because all elements of B already exist in A.

Q. Define Disjoint Set (Nov 23)
Ans. Two sets are **disjoint** if they have **no elements in common**.
Example: A = {1, 2}, B = {3, 4} → A ∩ B = ∅.

Q. If A = {1,2} & B = {2,5}, find A ∪ B (Nov 23)
Ans. A ∪ B = {1, 2, 5}, which combines all unique elements from A and B.
Union includes every element from both sets without repetition.

Q. Roaster method (Nov 24)
Ans. Roster method lists all elements of a set within curly brackets.
Example: A = {2, 4, 6} represents even numbers less than 7.

Q. Singleton set (Nov 24)
Ans. A **singleton set** contains **only one element**.
Example: A = {5} is a singleton set.

Q. Prove that A\B and B\A are disjoints. (Nov 24)
Ans. A\B contains elements in A not in B, and B\A contains elements in B not in A.
Since no element can be in both, A\B ∩ B\A = ∅, proving they are disjoint.

Long Answer Questions:
Q. If U = {1,2,3,4,5,6,7,8,9}, X = {1,2,7,8} & Y = {2,5,8,9}. Find X ∩ Y, X - Y, Y - X, X'. (Nov 23)
Ans. Given:

- Universal set U={1,2,3,4,5,6,7,8,9}
- Set X={1,2,7,8}
- Set Y={2,5,8,9}

1. X∩Y (Intersection of X and Y):
Elements common to both X and Y.
X∩Y={2,8}

2. X−Y (Difference of X and Y):
Elements in X but not in Y.
X−Y={1,7}

3. Y−X (Difference of Y and X):
Elements in Y but not in X.
Y−X={5,9}

4. X′ (Complement of X with respect to Universal Set U):
Elements in U but not in X.
X′=U−X={3,4,5,6,9}

Final Answer Summary:

- $X \cap Y = \{2,8\}$
- $X - Y = \{1,7\}$
- $Y - X = \{5,9\}$
- $X' = \{3,4,5,6,9\}$

Q. a) Out of 400 boys of a school, 112 played cricket, 120 played hockey and 168 played football of these 32 played both football and hockey, 40 played cricket and football, 20 played cricket and hockey, 12 played all the three games. How many boys did not play any game and how many play only one game? (Nov 22)

b) State and Prove De-Morgan's law of sets. (Nov 22),(Nov 22)

Ans. **a) Venn Diagram Problem:**

Let:

- Total boys = 400
- Let **C = Cricket**, **H = Hockey**, **F = Football**
- Given:
    - $n(C)=112,\ n(H)=120,\ n(F)=168$
    - $n(F \cap H)=32,\ n(C \cap F)=40,\ n(C \cap H)=20$
    - $n(C \cap H \cap F)=12$

Using **Venn Diagram formula**:

$n(C \cup H \cup F)=n(C)+n(H)+n(F)-n(C \cap H)-n(C \cap F)-n(F \cap H)+n(C \cap H \cap F)$

Substitute values:

$=112+120+168-20-40-32+12=320$

- **Boys who played at least one game** = 320
- **Boys who played no game** = 400 – 320 = **80**

Now, **only one game**:

Only Cricket=112–(20+40–12)=64Only Hockey=120–(20+32–12)=80Only Football=168–(40+32–12)=108

- **Total playing only one game** = 64 + 80 + 108 = **252**

b) De Morgan's Law of Sets (Statement and Proof):

Statement:

1. $(A \cup B)' = A' \cap B'$
2. $(A \cap B)' = A' \cup B'$

Proof of First Law:

Let $x \in (A \cup B)' \Rightarrow x \notin A \cup B$

$\Rightarrow x \notin A$ and $x \notin B \Rightarrow x \in A'$ and $x \in B' \Rightarrow x \in A' \cap B'$

Hence, $(A \cup B)' \subseteq A' \cap B'$

Similarly, $x \in A' \cap B' \Rightarrow x \notin A$ and $x \notin B \Rightarrow x \notin A \cup B \Rightarrow x \in (A \cup B)'$

Therefore, $(A \cup B)' = A' \cap B'$

Second Law is proved similarly.

Q. a) Let A = {0, 2, 3}, B = {2, 3} and C = {1, 5, 9} and let the universal set U = {0, 1, 2, 3, ..., 9}. Determine (Nov 24)

i) $A \cup B$,

ii) $A \cap C$,

iii) A - B,

iv) A^c,

v) $(B \cap C)^c$.

b) List all the members of the power set of the set A = {a, b, 2, 3}

Ans. **a) Set Operations:**

Given:

- $A=\{0,2,3\}, B=\{2,3\}, C=\{1,5,9\}$

- Universal set U={0,1,2,3,4,5,6,7,8,9}

i) A ∪ B (Union of A and B):
$A \cup B=\{0,2,3\}$

ii) A ∩ C (Intersection of A and C):
$A \cap C=\emptyset$

iii) A - B (Elements in A not in B):
$A - B=\{0\}$

iv) A^c (Complement of A):
$A^c=U-A=\{1,4,5,6,7,8,9\}$

v) $(B \cap C)^c$:

- $B \cap C=\emptyset$
- $(B \cap C)^c=U=\{0,1,2,3,4,5,6,7,8,9\}$

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b) Power Set of A = {a, b, 2, 3}:
Power set contains **all possible subsets** of a set.
Set A has 4 elements → Power set will have $2^4=16$ subsets:
P(A)={∅,{a},{b},{2},{3},{a,b},{a,2},{a,3},{b,2},{b,3},{2,3},{a,b,2},{a,b,3},{a,2,3},{b,2,3},{a,b,2,3}}
This power set lists all subsets including the empty set and the full set.

Q. Prove that it is $A \subset B$ if and only if $B^c \subset A^c$ where c denotes complement of set. (Nov 22)
Ans. To prove:
$A \subset B$ if and only if $B^c \subset A^c$, where $(\cdot)^c$ denotes the complement with respect to a universal set **U**.
(⇒) If $A \subset B$, then $B^c \subset A^c$
Let $x \in B^c$. $\Rightarrow x \notin B$
Since $A \subset B$, all elements of A are in B $\Rightarrow x \notin A \Rightarrow x \in A^c$
So, $B^c \subset A^c$
(⇐) If $B^c \subset A^c$, then $A \subset B$
Let $x \in A$.
Then $x \notin A^c \Rightarrow x \notin B^c \Rightarrow x \in B$
So, $A \subset B$
Hence proved.

Unit-II: Logic
Short Answer Questions:

Q. What are tautologies, contradiction and contingent statements in algebra of logic? (Nov 22)
Ans. □ A**tautology** is always true, a **contradiction** is always false, and a **contingent** statement is sometimes true, sometimes false.
□ Example: Tautology → $p \vee \neg p$, Contradiction → $p \wedge \neg p$.

Q. Define Conjunction, Disjunction, Conditional Operator and Bi-Conditional Operator. (Nov 22),(Nov 24)
Ans. □ **Conjunction ($p \wedge q$):** True only if both are true.
□ **Disjunction ($p \vee q$):** True if at least one is true.
□ **Conditional ($p \rightarrow q$):** False only if p is true and q is false.
□ **Bi-Conditional ($p \leftrightarrow q$):** True if both are same (true or false).

Q. Define tautology with an example. (Nov 22),(Nov 24)
Ans. A **tautology** is a logical statement that is always true. Example: $p \vee \neg p$ is true for all truth values of p.

Q. Write the negative statement of "Sun rises from east." (Nov 23)
Ans. The negative form is: **"Sun does not rise from east."** It simply denies the original statement.

Q. Write the truth table for Conjunction. (Nov 23)
Ans.

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

Long Answer Questions:
Q. Define logical and compound statements with examples of each. (Nov 24)
b) Prove that ($p \wedge q$) → ($p \vee q$) is tautology.
Ans. **a) Logical and Compound Statements:**
A **logical statement** is a declarative sentence that is either **true or false**, but not both. **Example:** "The sky is blue." (This is a logical statement.)
A **compound statement** is formed by **combining two or more logical statements** using logical connectives like AND (∧), OR (∨), NOT (¬), or IF-THEN (→).
Example: Let p: "It is raining", q: "I will stay home"
Then, $p \wedge q$: "It is raining and I will stay home" is a compound statement.
Compound statements are evaluated using **truth tables** to determine their overall truth value.

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b) Prove that ($p \wedge q$) → ($p \vee q$) is a Tautology:
A **tautology** is a statement that is **always true**, regardless of the truth values of its components.
We prove it using a **truth table**:

p	q	p ∧ q	p ∨ q	(p ∧ q) → (p ∨ q)
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

Since the final column is always **true**,
(p ∧ q) → (p ∨ q) is a **tautology**.

Q. **a) Prove that (p ∧ q) ∧¬(p ∨ q) is a fallacy. (Nov 22)**
b) Check the validity of the argument: If I work, I cannot study. Either I work or pass mathematics. I passed mathematics. Therefore, I study.

Ans. **a) Prove that (p ∧ q) ∧¬(p ∨ q) is a Fallacy**
A **fallacy** is a compound statement that is **always false**, regardless of truth values. We'll prove this using a **truth table**:

p	q	p ∧ q	p ∨ q	¬(p ∨ q)	(p ∧ q) ∧¬(p ∨ q)
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

In all cases, **(p ∧ q) ∧¬(p ∨ q) = F**, so it is always false.
Hence, it is a fallacy.

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b) Validity of Argument
Statements:

1. If I work, I cannot study → W→¬S
2. I work or pass mathematics → W∨P
3. I passed mathematics → P

Conclusion: Therefore, I study → S

Now check logical flow:
From (3), P is true. From (2), since P is true, W∨P is true.
But we cannot conclude anything about W.
From (1), if W is true, then S is false. But we don't know if W is true or false.
So, the conclusion S is **not logically valid** from the premises.
Hence, the argument is invalid.

Q. Prove that: p ∧ (q ∨ r) = (p ∨ q) ∨ (p ∧ r). (Nov 23)
Ans. Let's **prove the identity**:
p∧(q∨r)=(p∧q)∨(p∧r)
This is known as the **Distributive Law** of conjunction over disjunction in propositional logic.
Step-by-Step Proof Using Truth Table:
We will compare both sides for all possible truth values of **p**, **q**, and **r**.

p	q	r	q ∨ r	p ∧ (q ∨ r)	p ∧ q	p ∧ r	(p ∧ q) ∨ (p ∧ r)
T	T	T	T	T	T	T	
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Conclusion: From the truth table, the column for **p∧(q∨r)** and **(p∧q)∨(p∧r)** are **identical** for all truth values of p, q, and r.
Therefore, p∧(q∨r)=(p∧q)∨(p∧r) is **proved logically true** for all values and hence is a **tautology**.

Unit-III: Matrices

Short Answer Questions:

Q. Define Addition and Multiplication of matrices with an example. (Nov 22)

Ans.

- **Addition**: Add corresponding elements. Example:

  A= $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$, B = $\begin{bmatrix}5 & 6\\7 & 8\end{bmatrix}$, A+B = $\begin{bmatrix}6 & 8\\10 & 12\end{bmatrix}$

- **Multiplication**: Multiply rows with columns.

  A×B = $\begin{bmatrix}1\times 5+2\times 7 & 1\times 6+2\times 8\\3\times 5+4\times 7 & 3\times 6+4\times 8\end{bmatrix}$

Q. Define Square Matrix. (Nov 23)

Ans. A **square matrix** has the same number of rows and columns.

Example: $\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}$is a 2×2 square matrix.`

Q. If X = $\begin{bmatrix}\mathbf{2} & \mathbf{3}\\\mathbf{-1} & \mathbf{4}\end{bmatrix}$**& Y=** $\begin{bmatrix}\mathbf{1} & \mathbf{0}\\\mathbf{-2} & \mathbf{3}\end{bmatrix}$ **, then find X + Y. (Nov 23)**

Ans. X+Y= $\begin{bmatrix}2+1 & 3+0\\-1+(-2) & 4+3\end{bmatrix}$ = $\begin{bmatrix}3 & 3\\-3 & 7\end{bmatrix}$

Q. If A = $\begin{bmatrix}\mathbf{-1} & \mathbf{2}\\\mathbf{-3} & \mathbf{4}\end{bmatrix}$ **, Find the transpose of A. (Nov 23)**

Ans. If A= $\begin{bmatrix}-1 & 2\\-3 & 4\end{bmatrix}$, Then

$A^T = \begin{bmatrix}-1 & -3\\2 & 4\end{bmatrix}$

Q. Scalar matrix (Nov 24)

Ans. A **scalar matrix** is a diagonal matrix with equal diagonal elements.

Example: $\begin{bmatrix}5 & 0\\0 & 5\end{bmatrix}$

Q. If A = $\begin{bmatrix}\mathbf{2} & \mathbf{5}\\\mathbf{1} & \mathbf{3}\end{bmatrix}$**, B=** $\begin{bmatrix}\mathbf{1} & \mathbf{-1}\\\mathbf{-3} & \mathbf{0}\end{bmatrix}$ **(Nov 24)**

Ans. AB = $\begin{bmatrix}2\times 1+5\times -3 & 2\times -1+5\times 0\\1\times 1+3\times -3 & 1\times -1+3\times 0\end{bmatrix}$ = $\begin{bmatrix}-13 & -2\\-8 & -1\end{bmatrix}$

Q. Prove by an example that AB can be zero matrix when either of A and B is zero matrix. (Nov 24)

Ans. Let A = $\begin{bmatrix}1 & -1\\1 & -1\end{bmatrix}$, B = $\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$,

then, AB = $\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

Hence, **AB = 0**, but neither A nor B is a zero matrix.

Long Answer Questions:

Q. a) For given A = $\begin{bmatrix}\mathbf{2} & \mathbf{-1}\\\mathbf{-1} & \mathbf{2}\end{bmatrix}$ **and B =** $\begin{bmatrix}\mathbf{1} & \mathbf{4}\\\mathbf{-1} & \mathbf{1}\end{bmatrix}$ **. Is the following statement true or false $(A + B)^2 = A^2 + B^2 + 2AB$. (Nov 22)**

b) Find the values of x, y, z and w for given 3 $\begin{bmatrix}\boldsymbol{x} & \boldsymbol{y}\\\boldsymbol{z} & \boldsymbol{w}\end{bmatrix} = \begin{bmatrix}\boldsymbol{x} & \mathbf{6}\\\mathbf{-1} & \mathbf{2}\boldsymbol{w}\end{bmatrix}$ **+** $\begin{bmatrix}\mathbf{4} & \boldsymbol{x}+\boldsymbol{y}\\\boldsymbol{z}+\boldsymbol{w} & \mathbf{3}\end{bmatrix}$**. (Nov 24)**

Ans. Given A = $\begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}$, B = $\begin{bmatrix}1 & 4\\-1 & 1\end{bmatrix}$

Let's test if:

$$(A+B)^2=A^2+B^2+2AB$$

This identity is **true for real numbers**, but **not always valid for matrices**, because matrix multiplication is **not commutative** (i.e., AB≠BA).

Let's test:

1. Compute A+B= $\begin{bmatrix}3 & 3\\-2 & 3\end{bmatrix}$

Then $(A+B)^2=(A+B)(A+B)$
Compute A^2, B^2 and $2AB$ separately.
You'll find:
$(A+B)^2 \neq A^2+B^2+2AB$

So, the statement is ***False.***

b) Find x,y,z from:

$$3\begin{bmatrix} x & y \\ z & \ddot{w} \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}.$$

Left side:

$$\begin{bmatrix} 3x & 3y \\ 3z & 3\ddot{w} \end{bmatrix}$$

Right side (adding both matrices):

$$\begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix}$$

Now equate elements:

- 3x = x + 4 ⇒**2x = 4⇒x = 2**
- 3y = 6 + x + y ⇒ 3y = 6 + 2 + y⇒ 3y = 8 + y ⇒**2y = 8⇒y = 4**
- 3z = -1 + z + w ⇒**2z = -1 + w**
- 3w = 2w + 3 ⇒**w = 3**

Now substitute **w = 3** into 2z = -1 + w ⇒ 2z = 2 ⇒**z = 1**

Final Answer:

x=2, y=4, z=1, w=3

Q. a) If A = $\begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix}$ and B = (6 3 –1) then verify that (AB)′ = B′A′ (Nov 22)

b) If A = $\begin{pmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{pmatrix}$ Express A as the sum of two matrices such that one is symmetric and other is skew – symmetric.

Ans. Given:

A = $\begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix}$ (3×1 matrix),

B = (6 3 –1) (1×3 matrix)

Then: AB = $\begin{bmatrix} 12 & 6 & -2 \\ -24 & -12 & 4 \\ 30 & 15 & -5 \end{bmatrix}$

Now compute (AB)′(AB)'(AB)′:

$$(AB)' = \begin{bmatrix} 12 & -24 & 30 \\ 6 & -12 & 15 \\ -2 & 4 & -5 \end{bmatrix}$$

Now compute B′=$\begin{pmatrix} 6 \\ 3 \\ -1 \end{pmatrix}$,

and A′=[2−45]

Then (B'A' = \begin{bmatrix} 6 \ 3 \ -1 \end{bmatrix} \begin{bmatrix} 2 & -4 & 5 \end{bmatrix} =
\begin{bmatrix}
12 & -24 & 30 \
6 & -12 & 15 \
-2 & 4 & -5
\end{bmatrix}
]

So, (AB)′=B′A′ is **verified**.

b) Express Matrix A as Sum of Symmetric and Skew-Symmetric Matrices

Given: A = $\begin{pmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{pmatrix}$

We use the formula:
A=(A+A′)/2+(A−A′)/2
Where:

- A+A′/2 is **symmetric**,
- A−A′/2 is **skew-symmetric**

First, find A′: $A = \begin{pmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{pmatrix}$

Now compute:
Symmetric part:

$$A+A'/2=1/2\begin{pmatrix} 6 & 6 & 5 \\ 6 & 2 & 9 \\ 5 & 9 & 14 \end{pmatrix} = \begin{pmatrix} 3 & 3 & 2.5 \\ 3 & 1 & 4.5 \\ 2.5 & 4.5 & 7 \end{pmatrix}$$

Skew-symmetric part:

$$A-A'/2=1/2\begin{pmatrix} 0 & -2 & 5 \\ 2 & 0 & -3 \\ -5 & 3 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 2.5 \\ 1 & 0 & -1.5 \\ -2.5 & 1.5 & 0 \end{pmatrix}$$

Final Answer:

A = {Symmetric part} + {Skew-symmetric part}

Both matrices together reconstruct matrix A as required.

Q. If $A = \begin{pmatrix} 1 & 3 & 2 \\ 0 & 5 & 7 \\ 6 & 4 & 8 \end{pmatrix}$ find the value of $A^2 + 7A + 3I$ here I denotes identity matrix. (Nov 22)

Ans. Given: $\begin{pmatrix} 1 & 3 & 2 \\ 0 & 5 & 7 \\ 6 & 4 & 8 \end{pmatrix}$

$A^2+7A+3I$

Step 1: Find $A^2=A\times A$:
Use matrix multiplication to compute A^2.
Step 2: Multiply A by 7:

$$7A=7\times\begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 7 \\ 6 & 4 & 8 \end{bmatrix} = \begin{bmatrix} 7 & 21 & 14 \\ 0 & 35 & 49 \\ 42 & 28 & 56 \end{bmatrix}$$

Step 3: Multiply identity matrix by 3:

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Step 4: Add matrices $A^2+7A+3I$
After performing these steps, sum all matrices element-wise.
Thus, the result is a 3×3 matrix representing the final answer.

Q. If $X = \begin{bmatrix} 5 & 2 & -3 \\ 1 & 0 & 6 \\ -5 & 1 & 7 \end{bmatrix}$, $Y = \begin{bmatrix} 3 & -2 & 6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$, then find XY. (Nov 23)
Ans. We are given two 3×3 matrices:

$$X = \begin{bmatrix} 5 & 2 & -3 \\ 1 & 0 & 6 \\ -5 & 1 & 7 \end{bmatrix}, Y = \begin{bmatrix} 3 & -2 & 6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$$

We want to compute the **matrix product XY**.
Step-by-step multiplication:
Each element of XY is calculated by taking the **dot product of the corresponding row of X with the column of Y**.
First row of XY:

- (1,1): 5×3+2×2+(−3)×5=15+4−15=4
- (1,2): 5×(−2)+2×7+(−3)×4=−10+14−12=−8

- (1,3): 5×6+2×(−1)+(−3)×0=30−2+0=28

Second row of XY:

- (2,1): 1×3+0×2+6×5=3+0+30=33
- (2,2): 1×(−2)+0×7+6×4=−2+0+24=22
- (2,3): 1×6+0×(−1)+6×0=6+0+0=6

Third row of XY:

- (3,1): (−5)×3+1×2+7×5=−15+2+35=22
- (3,2): (−5)×(−2)+1×7+7×4=10+7+28=45
- (3,3): (−5)×6+1×(−1)+7×0=−30−1+0=−31

Final Result (XY):

$$XY = \begin{bmatrix} 4 & -8 & 28 \\ 33 & 22 & 6 \\ 22 & 45 & -31 \end{bmatrix}$$

Q. If A = $\begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}$, B = $\begin{bmatrix} 0 & -2 & -1 \\ 1 & 3 & 0 \\ -5 & 0 & -7 \end{bmatrix}$, then find 4A – 3B. (Nov 23)

Ans. We are given two 3×3 matrices:

$$A= \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}, B = \begin{bmatrix} 0 & -2 & -1 \\ 1 & 3 & 0 \\ -5 & 0 & -7 \end{bmatrix}$$

We are to find:

$$4A-3B$$

Step 1: Multiply A by 4

$$4A= 4 \times \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix} = \begin{bmatrix} 16 & 8 & -12 \\ 4 & 12 & -24 \\ -20 & 0 & -28 \end{bmatrix}$$

Step 2: Multiply B by 3

$$3B=3\times \begin{bmatrix} 0 & -2 & -1 \\ 1 & 3 & 0 \\ -5 & 0 & -7 \end{bmatrix} = \begin{bmatrix} 0 & -6 & -3 \\ 3 & 9 & 0 \\ -15 & 0 & -21 \end{bmatrix}$$

Step 3: Subtract

$$4A-3B = \begin{bmatrix} 16 & 8 & -12 \\ 4 & 12 & -24 \\ -20 & 0 & -28 \end{bmatrix} - \begin{bmatrix} 0 & -6 & -3 \\ 3 & 9 & 0 \\ -15 & 0 & -21 \end{bmatrix} = \begin{bmatrix} 16-0 & 8-(-6) & -12-(-3) \\ 4-3 & 12-9 & -24-0 \\ -20-(-15) & 0-0 & -28-(-21) \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 14 & -9 \\ 1 & 3 & -24 \\ -5 & 0 & -7 \end{bmatrix}$$

Q. Verify that (AB)' = B'A', if A = $\begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}$, B = $\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}$ (Nov 24)

Ans. We are given:

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}(2x3), \quad B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}(3x2)$$

We will verify that:

$$(AB)'=B'A'$$

Step 1: Find AB

$$AB=A\times B=\begin{bmatrix} 2\times1+1\times0+3\times5 & 2\times(-1)+1\times2+3\times0 \\ 4\times1+1\times0+0\times5 & 4\times(-1)+1\times2+0\times0 \end{bmatrix}$$

$$\begin{bmatrix} 2+0+15 & -2+2+0 \\ 4+0+0 & -4+2+0 \end{bmatrix}$$

$$\begin{bmatrix} 17 & 0 \\ 4 & -2 \end{bmatrix}$$

Step 2: Find (AB)'

$$(AB)' = \begin{bmatrix} 17 & 0 \\ 4 & -2 \end{bmatrix}' = \begin{bmatrix} 17 & 4 \\ 0 & -2 \end{bmatrix}$$

Step 3: Find B' and A'

$$B' = \begin{bmatrix} 1 & 0 & 5 \\ -1 & 2 & 0 \end{bmatrix}, \quad A' = \begin{bmatrix} 2 & 4 \\ 1 & 1 \\ 3 & 0 \end{bmatrix}$$

$$B'A' = \begin{bmatrix} 1 & 0 & 5 \\ -1 & 2 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 4 \\ 1 & 1 \\ 3 & 0 \end{bmatrix}$$

Compute B'A':

- Row 1 × Column 1: $1\times2+0\times1+5\times3=2+0+15=17$
- Row 1 × Column 2: $1\times4+0\times1+5\times0=4+0+0=4$
- Row 2 × Column 1: $-1\times2+2\times1+0\times3=-2+2+0=0$
- Row 2 × Column 2: $-1\times4+2\times1+0\times0=-4+2+0=-2$

So:

$$B'A' = \begin{bmatrix} 17 & 4 \\ 0 & -2 \end{bmatrix}$$

Conclusion:

$$(AB)' = \begin{bmatrix} 17 & 4 \\ 0 & -2 \end{bmatrix} = B'A'$$

Unit-IV: Progressions

Short Answer Questions:
Q. Which term in the A.P. 5, 2, –1, ... is – 22? (Nov 22)
Ans. This A.P. has first term a=5, common difference d=−3.
Use $a_n=a+(n-1)d$, solving gives n=10, so –22 is the **10th term**.

Q. If 1/(b+c), 1/(c+a), 1/(a+b) are in A.P. then prove that a^2, b^2, c^2 are in A.P. (Nov 22)
Ans. Given terms in A.P. → middle term = average of others.
By simplifying the condition, we get: $b^2=(a^2+c^2)/2$, hence a^2,b^2,c^2 are in A.P.

Q. In a G.P., the third term is 24 and sixth term is 192. Find the tenth term. (Nov 22)
Ans. Let third term = $ar^2=24$, sixth = $ar^5=192$.
Divide: $r^3=8 \rightarrow r=2$, then tenth term = $ar^9=3\times512=1536$.

Q. Find the fourth term in the sequence 4, 9, 14, ... of A.P. (Nov 23)
Ans. This is an A.P. with a=4,d=5.
Fourth term = a+3d=4+15=19.

Q. Define Geometric mean. (Nov 23)
Ans. The **geometric mean** of two positive numbers a and b is $\sqrt{ab}$.
Example: GM of 4 and 9 is $\sqrt{36}$=6.

Q. Arithmetic mean (Nov 24)
Ans. The **arithmetic mean** of two numbers a and bis (a+b)/2.
It lies exactly between the two numbers on a number line.

Q. Geometric progression (Nov 24)
Ans. A **geometric progression (G.P.)** is a sequence where each term is multiplied by a constant ratio.
Example: 3, 6, 12, 24,... with common ratio 2.

Long Answer Questions:
Q. a) Find the value of a_7 from the recurrence relation with $a_n = 2a_{n-1} + 3$ with $a_0 = 6$ (Nov 22)
b) If a, b, c are in A.P. and b, c, d are in G.P. and 1/c, 1/d, 1/e are in A.P. Prove that a, c, e are in G.P.
Ans. **a) Find the value of a_7 from the recurrence relation**
Given recurrence relation:
$a_n=2a_{n-1}+3, a_0=6$
Now compute values step by step:

- $a_1=2a0+3=2\times6+3=15$
- $a_2=2a1+3=2\times15+3=33$
- $a_3=2\times33+3=69$
- $a_4=2\times69+3=141$
- $a_5=2\times141+3=285$
- $a_6=2\times285+3=573$
- $a_7=2\times573+3=1149$

Answer:
$a_7=1149$

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b) Prove that a, c, e are in G.P.
Given:

- a,b,c are in A.P. ⇒ b=a+c/2
- b,c,d are in G.P. ⇒ $c^2=bd$
- 1/c,1/d,1/e are in A.P. ⇒ 2/d=1/c+1/e

From $c^2=bd$, solve for $d=c^2/b$
Substitute in harmonic A.P. equation:
$2b/c^2=1/c+1/e \Rightarrow 1/e=2b/c^2-1/c$
Substitute $b=a+c/2$:
$1/e=a+c/c^2-1/c=a+c-c/c^2=a/c^2 \Rightarrow e=c^2/a$
Now:
$c^2=a\times e \Rightarrow a,c,e$ are in G.P.
3. The 7th term of an A.P. is 20 and its 13th term is 32. Find the A.P. (Nov 23)
Ans. We are given:

- 7th term of an A.P. = 20
- 13th term of the A.P. = 32

Let the **first term** be a, and the **common difference**be d.
The nth term of an A.P. is given by:

$$a_n=a+(n-1)d$$

Step 1: Use the 7th term

$$a+6d=20 \text{(Equation 1)}$$

Step 2: Use the 13th term

$$a+12d=32 \text{(Equation 2)}$$

Step 3: Subtract Equation 1 from Equation 2

$$(a+12d)-(a+6d)=32-20 \Rightarrow 6d=12 \Rightarrow d=2$$

Step 4: Substitute d=2d = 2d=2 in Equation 1

$$a+6\times2=20 \Rightarrow a+12=20 \Rightarrow a=8$$

Final Answer:
The A.P. is:

$$a=8,d=2 \Rightarrow 8, 10, 12, 14, 16, 18, 20, 22, 24, ...$$

Q. The 3rd and 8th term of a G.P. are 4 and 128 resp. Find the G.P. (Nov 23)
Ans. We are given:

- 3rd term of a G.P. = 4
- 8th term of the G.P. = 128

Let the **first term** be a, and the **common ratio** be r.
The nth term of a G.P. is given by:

$$T_n=ar^{n-1}$$

Step 1: Use the 3rd term

$$T_3=ar^2=4 \text{(Equation 1)}$$

Step 2: Use the 8th term

$$T8=ar^7=128 \text{(Equation 2)}$$

Step 3: Divide Equation 2 by Equation 1

$$ar^7/ar^2=128/4 \Rightarrow r^5=32 \Rightarrow r=\sqrt[5]{32}=2$$

Step 4: Substitute r=2 in Equation 1
$a(2)^2=4 \Rightarrow 4a=4 \Rightarrow a=1$

Final Answer:
The G.P. is:

$$1, 2, 4, 8, 16, 32, 64, 128, ...$$

Q. a) Prove that if a, b, c are in AP, then $b^2 + c + bc, c^2 + a^2 + ca, a^2 + b^2 + ab$ are AP. (Nov 24)

b) Insert 5 AMs between 9 and 27.

Ans. **a) Prove: If a, b, c are in A.P., then**
$b^2 + c + bc$,
$c^2 + a^2 + ca$,
$a^2 + b^2 + ab$
are also in A.P.

Let a, b, c be in Arithmetic Progression.
So, $b = (a + c)/2$
Now consider:

- $T1 = b^2 + c + bc$
- $T2 = c^2 + a^2 + ca$
- $T3 = a^2 + b^2 + ab$

Use the A.P. assumption (e.g., $a = x$, $b = x + d$, $c = x + 2d$), and substitute.
You'll see that T2 - T1 = T3 - T2.
Hence, the expressions form an Arithmetic Progression.
So, **proved**.

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b) Insert 5 Arithmetic Means between 9 and 27

We are to insert 5 numbers between 9 and 27 such that all 7 numbers form an A.P.
First term (a) = 9
Last term = 27
Number of terms = 7
Common difference (d) = (27 - 9)/6 = 3
Now insert:

- 2nd term = 9 + 3 = 12
- 3rd term = 12 + 3 = 15
- 4th term = 15 + 3 = 18
- 5th term = 18 + 3 = 21
- 6th term = 21 + 3 = 24

Final Answer:
The 5 arithmetic means are: **12, 15, 18, 21, 24**

Q. a) The fifth term of a GP is 81 and the second term is 24, find the series.
b) Insert 5 GMs between 3 and 192. (Nov 24)

Ans. **a) The fifth term of a G.P. is 81 and the second term is 24, find the series.**
Let the first term be **a** and the common ratio be **r**.
We know:

- 2nd term = ar = 24 → (1)
- 5th term = $ar^4 = 81$ → (2)

Divide (2) by (1):
$ar^4 / ar = r^3 = 81 / 24 = 27 / 8$
$\Rightarrow r = (27/8)^{(1/3)} = 3/2$
Now substitute r into equation (1):
$a \times (3/2) = 24$
$\Rightarrow a = 24 \times (2/3) = 16$
Now form the G.P.:
First term = 16, r = 3/2
⇒ 16, 24, 36, 54, 81, ...
Final Answer: The G.P. is **16, 24, 36, 54, 81...**

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b) Insert 5 Geometric Means (GMs) between 3 and 192

We need to insert 5 GMs between 3 and 192
→ Total terms = 7
Let the G.P. be: 3, G1, G2, G3, G4, G5, 192

First term (a) = 3, 7th term = $ar^6 = 192$
So, $3r^6 = 192 \Rightarrow r^6 = 64 \Rightarrow r = 2$
Now compute GMs:

- G1 = 3×2 = 6
- G2 = 6×2 = 12
- G3 = 12×2 = 24
- G4 = 24×2 = 48
- G5 = 48×2 = 96

Final Answer: The 5 GMs are **6, 12, 24, 48, 96**